Wolfram|Alpha Step-by-Step Solution

Wolfram|Alpha Input: integrate (x-1)/(√x-1)

STEP 1

Take the integral:

$$\int \frac{x-1}{\sqrt{x}-1}\,dx$$

STEP 2

For the integrand $\frac{x-1}{\sqrt{x}-1}$, substitute $u = \sqrt{x}$ and $du = \frac{1}{2\sqrt{x}}\,dx$:

$$= 2\int \frac{u\,(u^2-1)}{u-1}\,du$$

STEP 3

For the integrand $\frac{u\,(u^2-1)}{u-1}$, cancel common terms in the numerator and denominator:

$$= 2\int u\,(u+1)\,du$$

STEP 4

For the integrand $u\,(u+1)$, substitute $s = u+1$ and $ds = du$:

$$= 2\int (s-1)\,s\,ds$$

STEP 5

Expanding the integrand $(s-1)\,s$ gives $s^2 - s$:

$$= 2\int (s^2 - s)\,ds$$

STEP 6

Integrate the sum term by term and factor out constants:

$$= 2\int s^2\,ds - 2\int s\,ds$$

STEP 7

The integral of s^2 is $\frac{s^3}{3}$:

$$= \frac{2\,s^3}{3} - 2\int s\,ds$$

STEP 8

The integral of s is $\frac{s^2}{2}$:

$$= \frac{2\,s^3}{3} - s^2 + \text{constant}$$

STEP 9

Substitute back for $s = u + 1$:

$$= \frac{2}{3}(u+1)^3 - (u+1)^2 + \text{constant}$$

STEP 10

Substitute back for $u = \sqrt{x}$:

$$= \frac{2}{3}\left(\sqrt{x}+1\right)^3 - \left(\sqrt{x}+1\right)^2 + \text{constant}$$

STEP 11

Factor the answer a different way:

$$= \frac{2x^{3/2}}{3} + x - \frac{1}{3} + \text{constant}$$

STEP 12

Which is equivalent for restricted x values to:

Answer:

$$= \frac{2x^{3/2}}{3} + x + \text{constant}$$